Learning the Structure of Probabilistic Sentential Decision Diagrams

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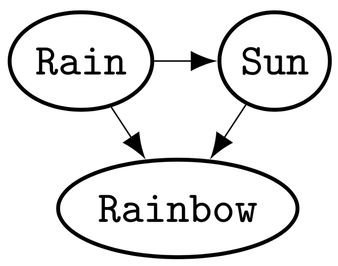
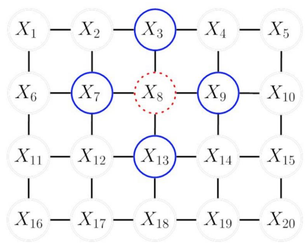
Background

Tractable vs. Intractable Representation

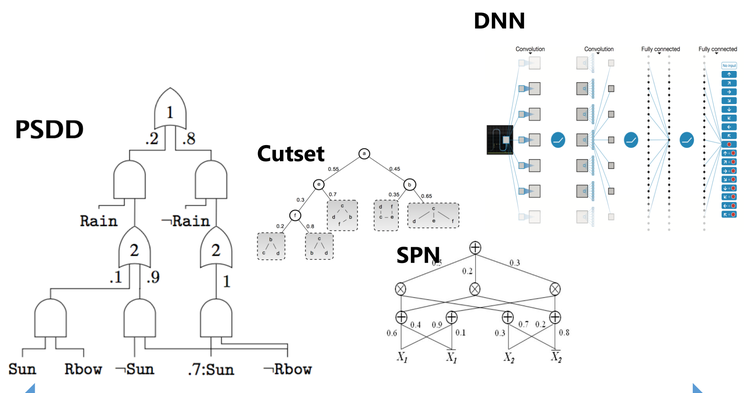
Bayesian networks



Markov networks



Tradeoff: Properties vs. Expressiveness



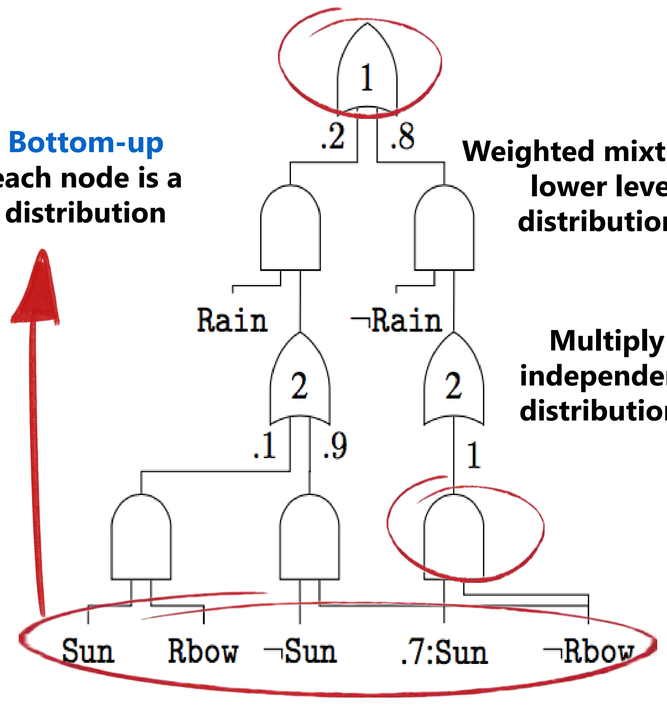
PSDD

The Most Powerful Circuit Representation

- Closed-form parameter estimation
- Efficient multiplication
- Fast MPE inference
- Etc.

Structure learning

Bottom-up
each node is a distribution



Weighted mixture of lower level distributions

Multiply independent distributions

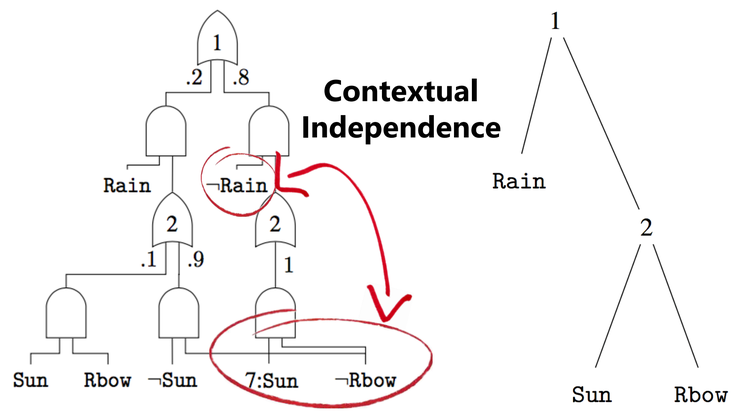
$$\Pr(\texttt{Rain}) = 0.2,$$

$$\Pr(\texttt{Sun} \mid \texttt{Rain}) = \begin{cases} 0.1 \text{ if } \texttt{Rain} \\ 0.7 \text{ if } \neg\texttt{Rain} \end{cases}$$

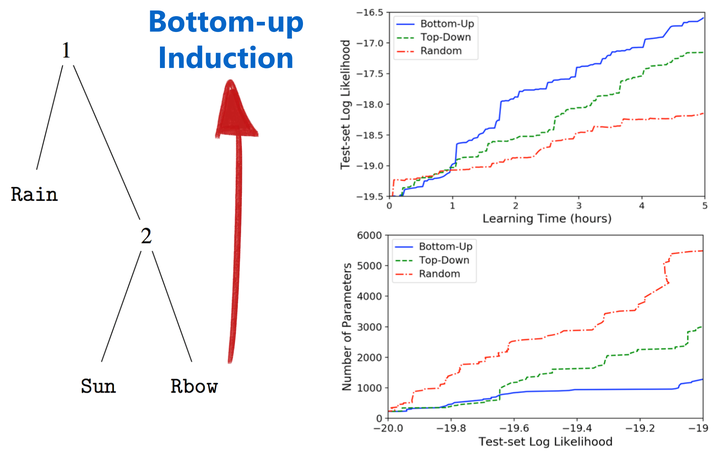
$$\Pr(\texttt{Rbow} \mid \texttt{R}, \texttt{S}) = \begin{cases} 1 \text{ if } \texttt{Rain} \wedge \texttt{Sun} \\ 0 \text{ otherwise} \end{cases}$$

Are PSDDs amenable to tractable learning

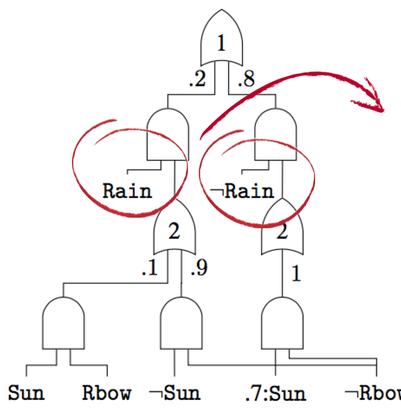
Contextual Independence & Vtree



Bottom-up Induction

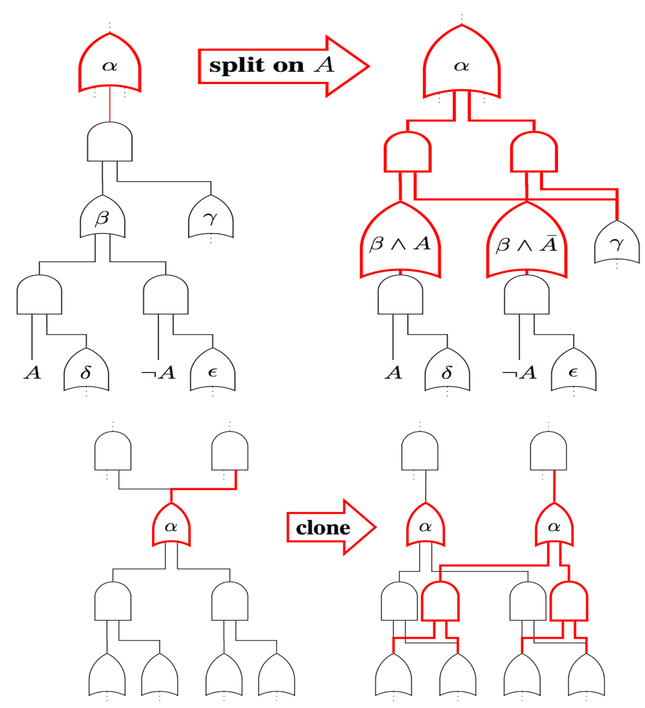


Determinism



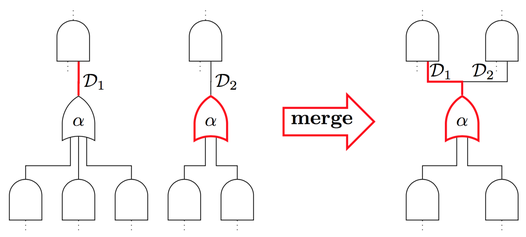
Branching over sentences on prime variables

Search for Structure (LearnPSDD)



$$\text{score} = \frac{\ln \mathcal{L}(r' \mid \mathcal{D}) - \ln \mathcal{L}(r \mid \mathcal{D})}{\text{size}(r') - \text{size}(r)}$$

Future Work (MergePSDD)



Experiments

Roadmap

1. Vtree learning
2. Construct the most naïve PSDD
3. LearnPSDD (search for better structure)

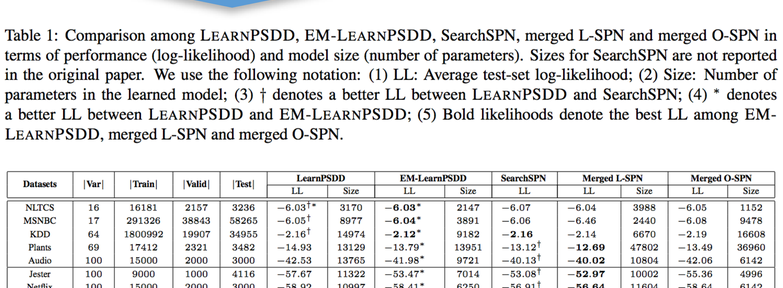
LearnPSDD

Table 1: Comparison among LearnPSDD, EM-LearnPSDD, SearchSPN, merged L-SPN and merged O-SPN in terms of performance (log-likelihood) and model size (number of parameters). Sizes for SearchSPN are not reported in the original paper. We use the following notation: (1) LL: Average test-set log-likelihood; (2) Size: Number of parameters in the learned model; (3) † denotes a better LL between LearnPSDD and SearchSPN; (4) * denotes a better LL between LearnPSDD and EM-LearnPSDD; (5) Bold likelihoods denote the best LL among EM-LearnPSDD, merged L-SPN and merged O-SPN.

Datasets	|Var|	|Train|	|Valid|	|Test|	LearnPSDD LL	LearnPSDD Size	EM-LearnPSDD LL	EM-LearnPSDD Size	SearchSPN LL	Merged L-SPN LL	Merged L-SPN Size	Merged O-SPN LL	Merged O-SPN Size
NLTCS	16	16181	2157	3236	−6.03†*	3170	**−6.03***	2147	−6.07	−6.04	3988	−6.05	1152
MSNBC	17	291326	38843	58265	−6.05†	8977	**−6.04***	3891	−6.06	−6.46	2440	−6.08	9478
KDD	64	1800992	19907	34955	−2.16†	14974	**−2.12***	9182	**−2.16**	−2.14	6670	−2.19	16608
Plants	69	17412	2321	3482	−14.93	13129	−13.79*	13951	−13.12†	**−12.69**	47802	−13.49	36960
Audio	100	15000	2000	3000	−42.53	13765	−41.98*	9721	−40.13†	**−40.02**	10804	−42.06	6142
Jester	100	9000	1000	4116	−57.67	11322	−53.47*	7014	−53.08†	**−52.97**	10002	−55.36	4996
Netflix	100	15000	2000	3000	−58.92	10997	−58.41*	6250	−56.91†	**−56.64**	11604	−58.64	6142
Accidents	111	12758	1700	2551	−34.13	10489	−33.64*	6752	−30.02†	**−30.01**	13322	−30.83	6846
Retail	135	22041	2938	4408	−11.13	4091	**−10.81***	7251	−10.97†	−10.87	2162	−10.95	3158
Pumsb-Star	163	12262	1635	2452	−34.11	10489	−33.67*	7965	−28.69†	**−24.11**	17604	−24.34	18338
DNA	180	1600	400	1186	−89.11*	6068	−81.76†	14864	−81.76†	**−85.51**	4320	−87.49	1430
Kosarek	190	33375	4450	6675	−10.99†	11034	−10.81*	10179	−11.00	**−10.62**	5318	−10.98	6712
MSWeb	294	29441	32750	5000	−10.18†	11389	−9.97*	14512	−10.25	**−9.90**	16484	−10.06	12770
Book	500	8700	1159	1739	−35.90	15197	−34.97*	11292	−34.91†	**−34.76**	11998	−37.44	11916
EachMovie	500	4524	1002	591	−56.43*	12483	−58.01	16074	−53.28†	**−52.07**	15998	−58.05	19846
WebKB	839	2803	558	838	−163.42	10033	−161.09*	18431	−157.88†	**−153.55**	20134	−161.17	10046
Reuters-52	889	6532	1028	1530	−94.94	10585	−89.61*	9546	−86.38†	**−83.90**	46232	−87.49	28334
20NewsGrp.	910	11293	3764	3764	−161.41	12222	−161.09*	18431	−153.63†	**−154.67**	43684	−161.46	29016
BBC	1058	1670	225	330	−260.83	10585	**−253.19***	20327	−252.13†	−253.45	21160	−260.59	8454
AD	1556	2461	327	491	−30.49*	9666	−31.78	9521	−16.97†	−16.77	49790	**−15.39**	31070

Compare with O-SPN: smaller size in 14, better LL in 11, win on both in 6

Compare with L-SPN: smaller size in 14, better LL in 6, win on both in 2

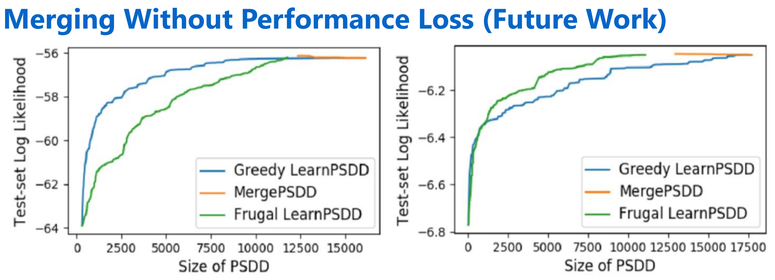


EM/Bagging

Datasets	Var	LearnPSDD Ensemble	Best-to-Date
NLTCS	16	−5.99†	−6.00
MSNBC	17	−6.04†	−6.04†
KDD	64	−2.11†	−2.12
Plants	69	−13.02	−11.99†
Audio	100	−39.94	−39.49†
Jester	100	−51.29	−41.11†
Netflix	100	−55.71†	−55.84
Accidents	111	−30.16	−24.87†
Retail	135	−10.72†	−10.78
Pumsb-Star	163	−26.12	−22.40†
DNA	180	−88.01	−80.03†
Kosarek	190	−10.52†	−10.54
MSWeb	294	−9.89	−9.22†
Book	500	−34.97	−30.18†
EachMovie	500	−58.01	−51.14†
WebKB	839	−161.09	−150.10†
Reuters-52	889	−89.61	−80.66†
20NewsGrp.	910	−155.97	−150.88†
BBC	1058	−253.19	−233.26†
AD	1556	−31.78	−14.36†

State-of-the-art in 6 datasets

Merging Without Performance Loss (Future Work)



Retain the ability to fit logically constrained distributions

Discrete Multi-valued Data

$\boldsymbol{A}$: a_1, a_2, a_3

$$\boldsymbol{A_1, A_2, A_3}$$
$$(\boldsymbol{a_1} \vee \boldsymbol{a_2} \vee \boldsymbol{a_3}) \wedge \begin{cases} \boldsymbol{a_1} \wedge \neg\boldsymbol{a_2} \wedge \neg\boldsymbol{a_3} \\ \vee \\ \neg\boldsymbol{a_1} \wedge \boldsymbol{a_2} \wedge \neg\boldsymbol{a_3} \\ \vee \\ \neg\boldsymbol{a_1} \wedge \neg\boldsymbol{a_2} \wedge \boldsymbol{a_3} \end{cases}$$

1. Compile logics into a SDD
2. Convert to a PSDD: Parameter estimation
3. LearnPSDD

Datasets	No Constraint	PSDD	LearnPSDD
Adult	−18.41	−14.14	−12.86
CovType	−14.39	−8.81	−7.32

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