# Ordering-Based Search for Tractable Bayesian Networks

Jessa Bekker, Guy Van den Broeck, Jesse Davis
jessa.bekker@cs.kuleuven.be

## Goal and Motivation

Upgrading ordering-based search [1] for learning tractable Bayesian networks.

- For Bayesian networks, this works better than structure-search
- Flexible tractable representation (SDD [2]) makes this possible

## Tractable learning

Structure Learning of Bayesian networks that:

- Guarantee efficient inference for certain queries
- Guarantee exact reasoning

Simultaneously learn 1) a Bayesian network
and 2) a **tractable representation** for it.

Incrementally change the network so that:

- The accuracy improves
- Querying remains efficient
  (= keep tractable representation small)

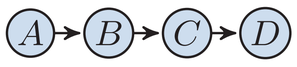
Possible tractable representations:

**Arithmetic Circuit (AC):** Not flexible: cannot execute *swap*.
*Used by ACBN [3], only other tractable BN learner.*

**Sentential Decision Diagram (SDD):** Flexible! We use this.

## Ordering-based search

- Ordering over variables in network

  E.g: 
  

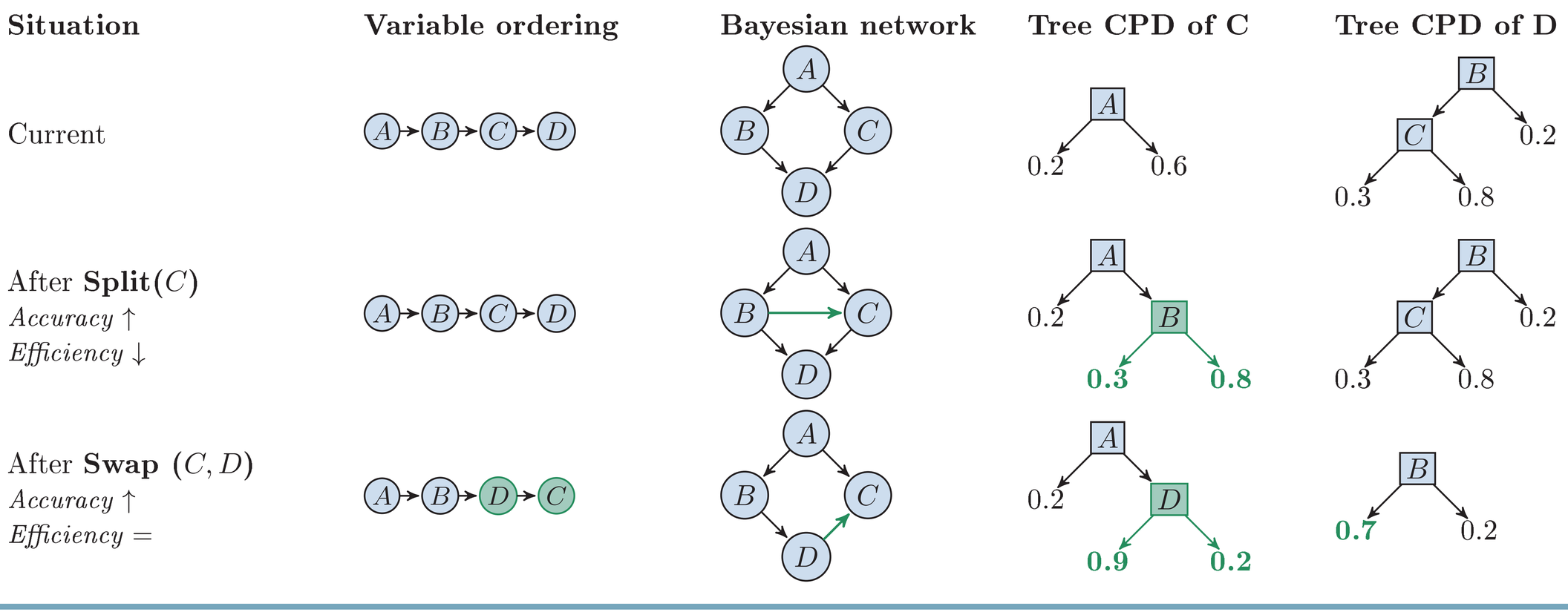
  possible parents of $C$: $\{A, B\}$
- Given ordering ⇒ finding the best network is easy
- Search over orderings by **swapping** neighbors in ordering
- Optimizations:
  - Caching
  - Random restarts
  - Sparse candidates

## Challenges in tractable context

1. Full CPTs are not tractable
   ⇒ Use **decision tree CPDs**
2. Score is not decomposable
   Score function has two parts:
   - Likelihood
   - Efficiency = # edges in SDD

   **Efficiency is not decomposable!**
   ⇒ Fix efficiency when swapping
   ⇒ Add *split* operator that adds a split to tree CPD

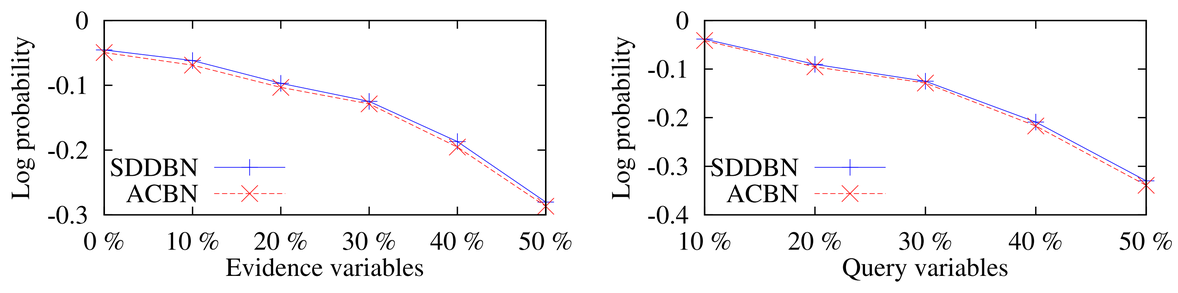
## Search operators




## Preliminary results

Tested with conditional probability queries $\Pr(\mathbf{X}|\mathbf{Y})$
$\mathbf{X}$: Query variables
$\mathbf{Y}$: Evidence variables

The queries are generated from test data




SDDBN and ACBN yield similar results.

## Ongoing work

- Datasets with more variables
- Reduce local minima
  - Good splits below a bad split are never reached
  -  , $B$ and $C$ not in ech other's sparse candidate set ⇒ Cannot reach better ordering 

## Related work

[1] M. Teyssier and D. Koller. Ordering-based search: A simple and effective algorithm for learning bayesian networks. *UAI*, 2005.

[2] A. Darwiche. SDD: A new canonical representation of propositional knowledge bases. *IJCAI*, 2011.

[3] D. Lowd and P. Domingos. Learning arithmetic circuits. *UAI*, 2008.